A two-phase geothermal model with fracture network and multi-branch wells for geothermal simulation

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Collaboration and Funding

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Outline

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1. **Motivation and Previous Projects**
   - i) Geothermal energy.
   - ii) Simulation of geothermal systems.
2. **Theoretical and Dicretization Frameworks**
   - i) Two-phase reservoir model.
   - ii) Two-phase well model.
   - iii) The discrete nonlinear system.
3. **Numerical Tests**
   - i) Diphasic test case in a volcanic zone.
     Ref: [Armandine Les Landes, CQ, Jeannin, Lopez, Masson 2021]

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Geothermal Energy

Exploitation of geothermal energy

- **Objective**: To recover the heat naturally present underground and in groundwater and use it as a source of energy.
- **Advantages**:
  - Clean energy source.
  - Renewable energy.
  - Depth = heat!
- **Restrictions**:
  - Local energy.
  - Reservoirs may become depleted/consumed as they are exploited.

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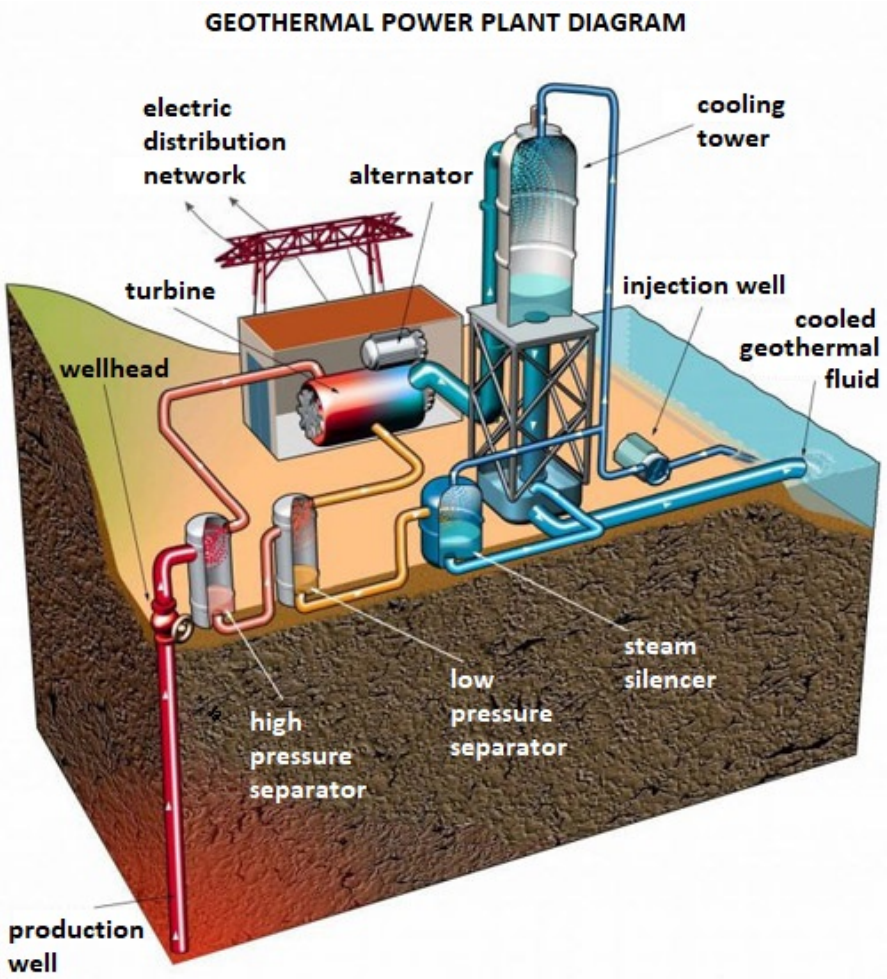
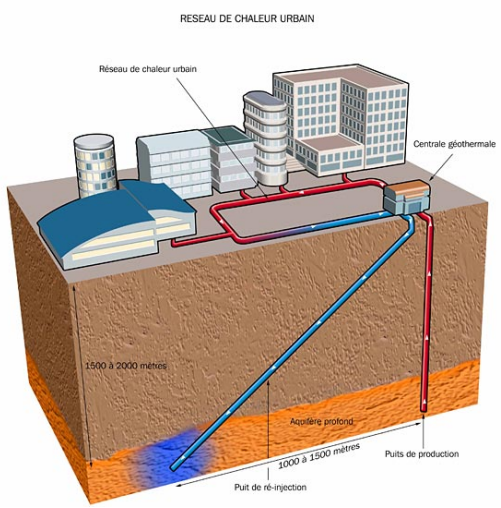
Geothermal Energy

Exploitation of geothermal energy- How geothermal energy is produced?





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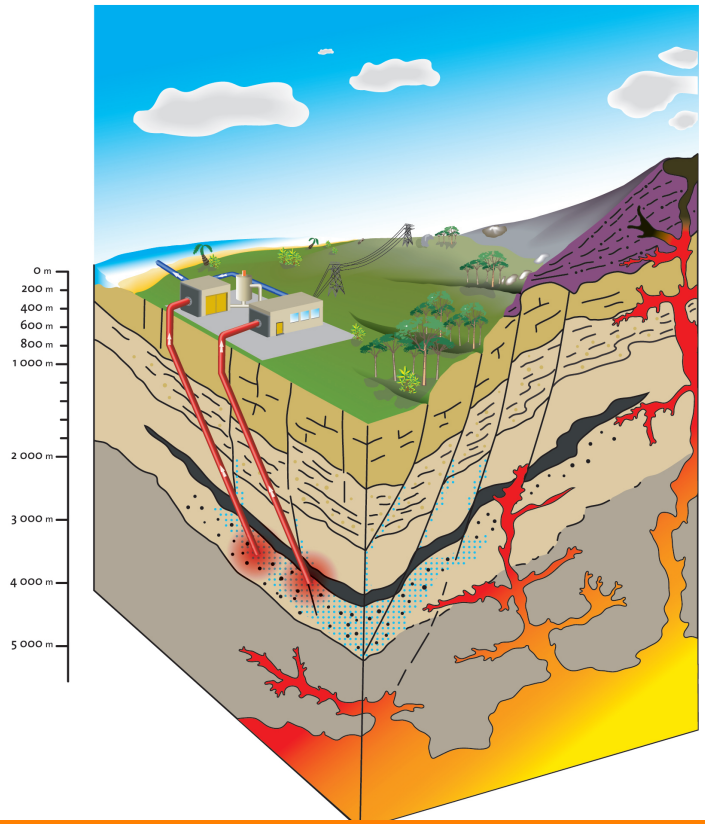
Geothermal Energy

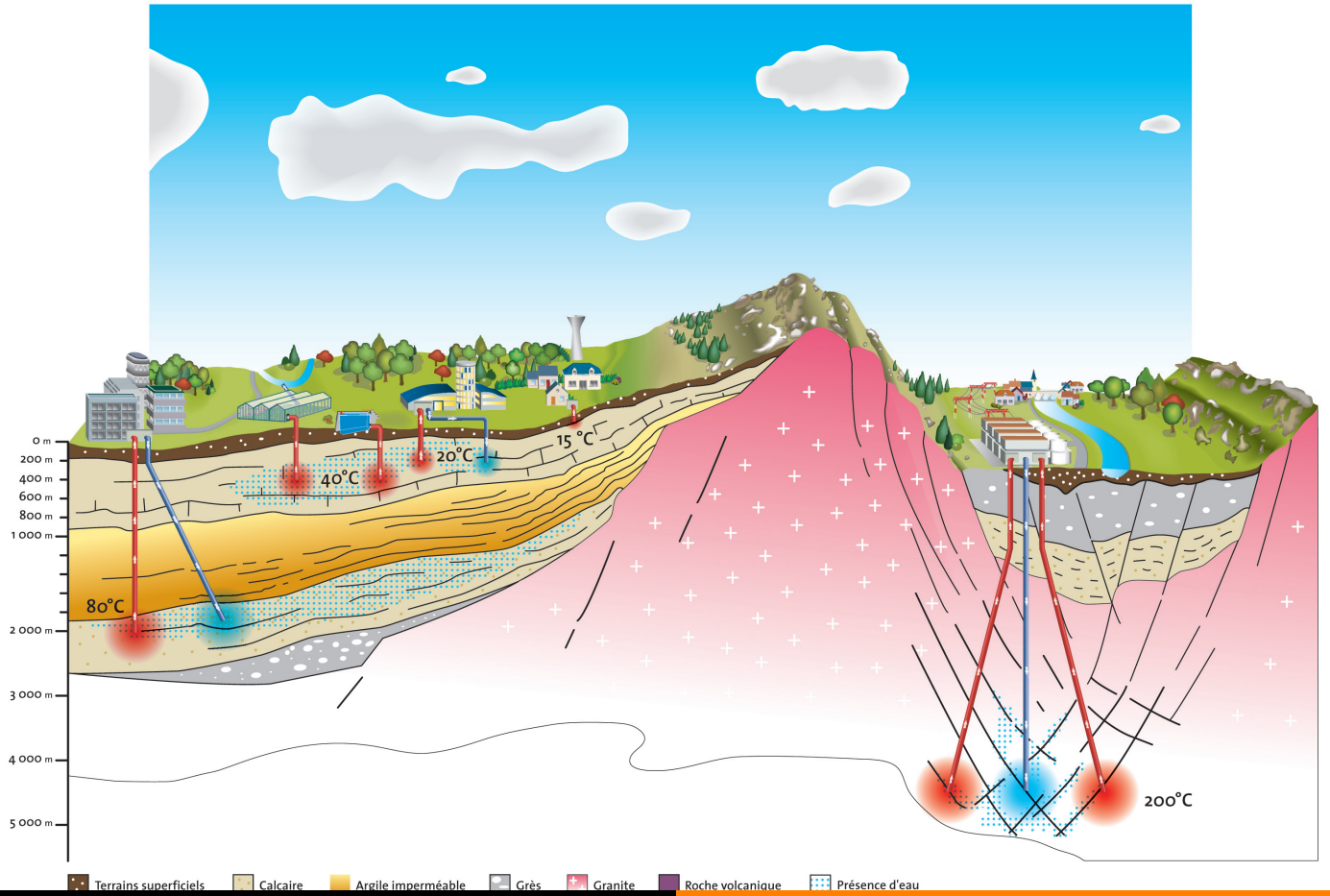
Exploitation of geothermal energy

- High-temperature geothermal energy (from 90 to 150°C) as in the **Bouillante power plant in Guadeloupe**.
  - Energy production since 1986 when BRGM drilled the initial wells.
  - Currently supplying about 6% of electricity.



Geothermal Energy

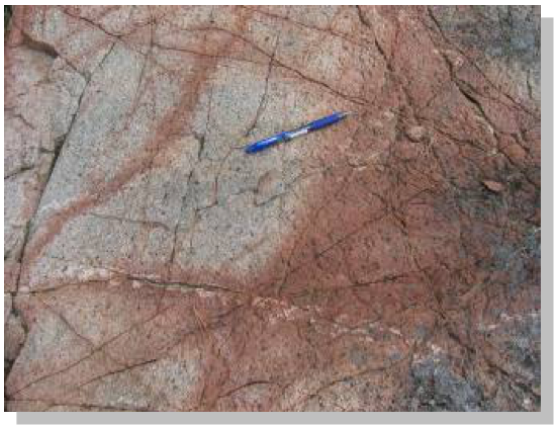
Exploitation of geothermal energy- How geothermal energy is produced?



Geothermal Energy

Exploitation of geothermal energy

- Important to consider geology faults and fractures.

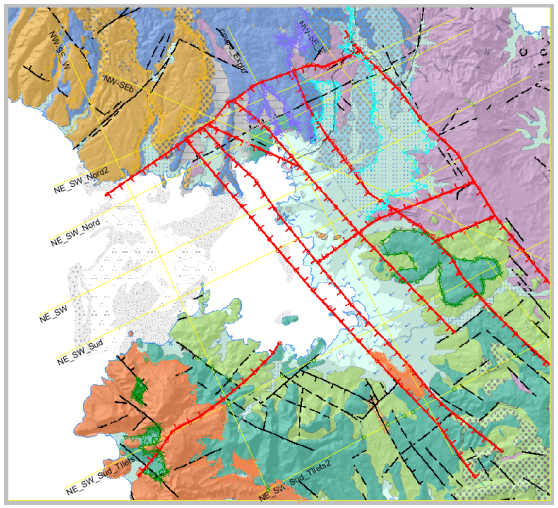


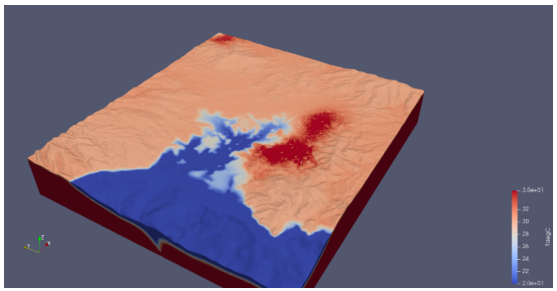
Simulation of Geothermal Systems

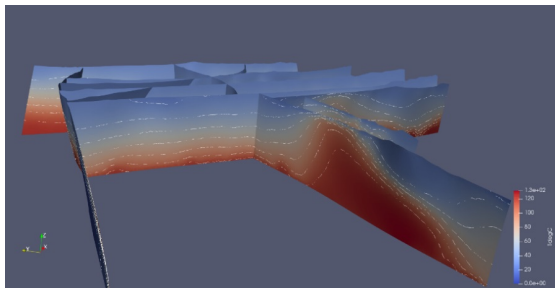
Simulation of Geothermal Systems

- **The Code ComPASS.**
  - Developed since 2012.
  - The ANR CHARMS project: BGRM, LJAD-Inria, Storengy, la Maison de la Simulation and the Jacques Louis Lions lab.
  - First publication: [Xing, Masson, Lopez 2017].

The Lamentin bay, Martinique. Surface/faults map, and a mesh discretization using ComPASS:







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Simulation of Geothermal Systems

Simulation of Geothermal Systems

- **The Code ComPASS.**
  - Co-developed and managed by BRGM.
  - Full industrial code:
    - Developed in layers using Fortran90 (core) ->C++/17 (wrappers) -> Python3 (user level).
    - Updated daily using Git on Inria-GitLab.
    - Parallel code using MPI/PETSC/METIS...
    - Work environment using Docker.

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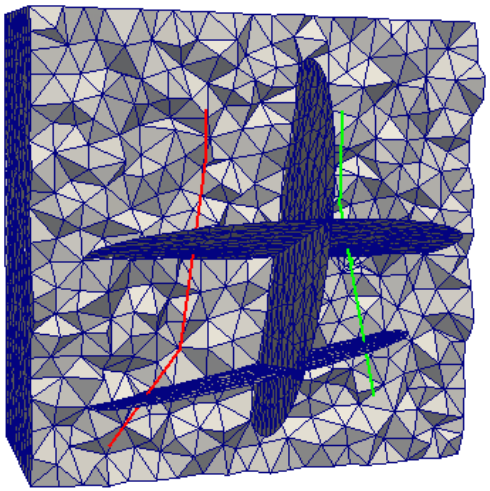
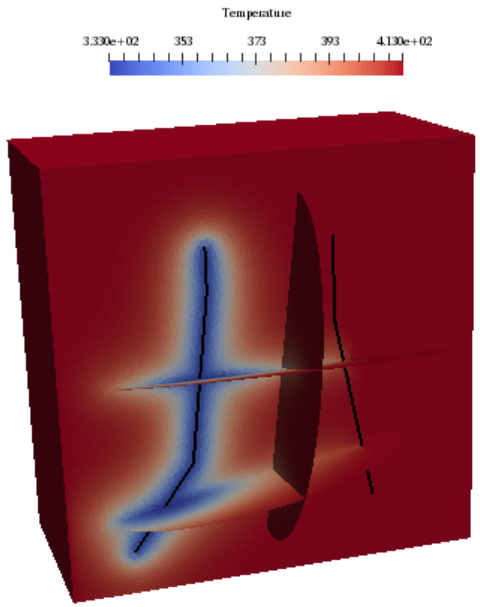
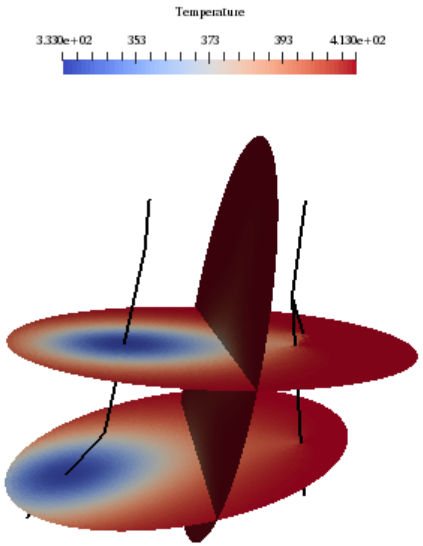
Simulation of Geothermal Systems

Simulation of Geothermal Systems

- **The Code ComPASS**- A particular test case with wells.
  - One component (water-h_2o), single-phase, one producer well, one producer injection, and some fractures.
  - Test case from [Beaude *et. al.* 2018].







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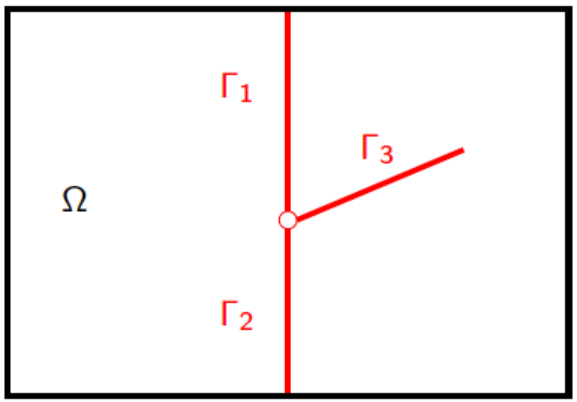
Simulation of Geothermal Systems

- **First objective of the current project**: Extend the model and its implementation in ComPASS to two phases. In particular, liquid water and vapor.

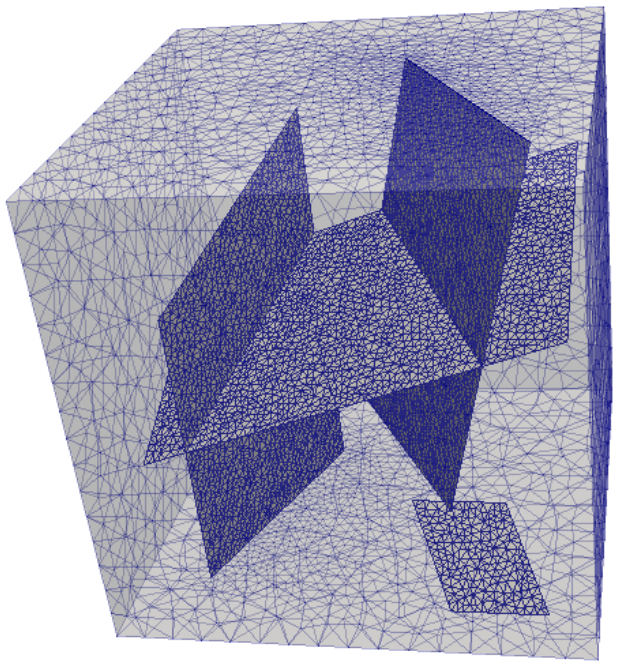
Theoretical Framework

Model-Topology

- Let Ω denote a bounded domain of $\mathbb{R}^3$ assumed to be polyhedral.
- The fractures are represented as interfaces of codimension 1.



Example of a 2D domain with 3 intersecting fractures $\Gamma_1, \Gamma_2, \Gamma_3$.



Example in 3D.

Theoretical Framework

Model-H_2O-Liquid (l) and vapor (g)

- **Unknowns**: Pressure: p_m. Temperature: T_m. Saturations: s_m^l, s_m^g.
- The set of equations couples the **mass**, **energy**, **volume** balance equations and the **thermodynamic equilibrium** in the matrix (subscript $_m$):

$$
\begin{cases}
\phi_m \, \partial_t \Big(\sum\limits_{\alpha \in \{\ell, \mathrm{g}\}} \rho^\alpha(p_m, T_m) s_m^\alpha \Big) + \mathrm{div}(\mathbf{q}_m^{\mathrm{h_2o}}) = 0, \\
\phi_m \, \partial_t \Big(\sum\limits_{\alpha \in \{\ell, \mathrm{g}\}} \rho^\alpha(p_m, T_m) e^\alpha(p_m, T_m) s_m^\alpha \Big) + (1 - \phi_m) \partial_t E_r(p_m, T_m) + \mathrm{div}(\mathbf{q}_m^e) = 0, \\
\sum\limits_{\alpha \in \{\ell, \mathrm{g}\}} s_m^\alpha = 1, \\
p_m = p_{\mathrm{sat}}(T_m) \text{ if } s_m^l > 0 \text{ and } s_m^g > 0, \\
s_m^g = 0 \text{ if } p_m > p_{\mathrm{sat}}(T_m), \\
s_m^l = 0 \text{ if } p_m < p_{\mathrm{sat}}(T_m),
\end{cases}
$$

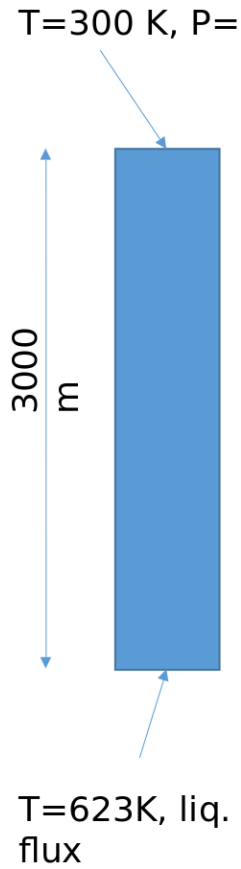
where ϕ_m: porosity, ρ^α: mass density, $\mathbf{q}_m^{\mathrm{h_2o}}$: mass flux, $\mathbf{q}_m^e$: energy flux, e: specific internal energy, E_r: rock energy density, and p_{sat}: the saturated pressure.

- The mass flux $\mathbf{q}_m^{\mathrm{h_2o}}$ computed using Darcy law.
- The energy flux $\mathbf{q}_m^e$ computed using convection and the Fourier law.
- Similarly for fractures, i.e., Pressure p_f. Temperature: T_f . . .

Theoretical Framework

Model-H_2O-Liquid (l) and vapor (g)

- **Simple example**: Water column and heat transfer of energy by **thermal convection**.



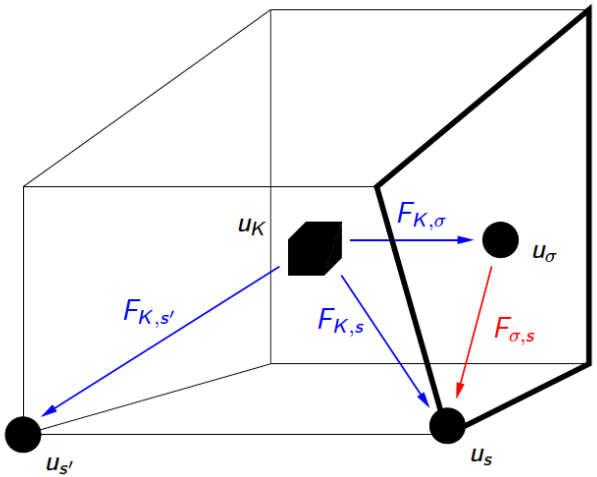
Discretization

Discretization: The VAG Scheme (Notation and DOFs)

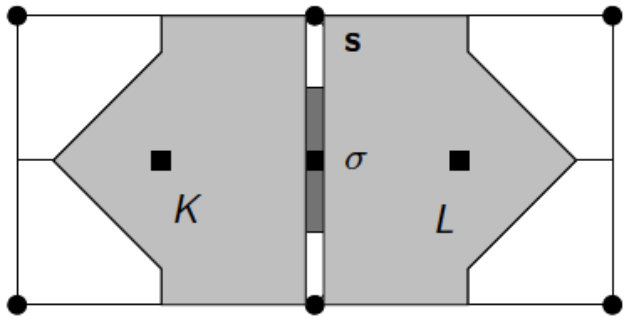
Let $\mathcal{M}$ be the set of cells of the polyhedral mesh of Ω.
The set of faces of the mesh is $\mathcal{F}$ and $\mathcal{F}_K$ is the set of faces of the cell $K \in \mathcal{M}$.
The set of vertices of the mesh is $\mathcal{V}$ and $\mathcal{V}_\sigma$ is the set of vertices of the face σ.
Let $\mathcal{V}_K$ be the set of vertices of the cell K.
Let $\mathcal{F}_\Gamma$ the set of fracture faces (the mesh is conforming w.r.t. the fracture network).

- Degrees of Freedom (DOFs) of the VAG Scheme:

$$V_\mathcal{D} = \{v_K, v_\mathbf{s}, v_\sigma \in \mathbb{R}, K \in \mathcal{M}, \mathbf{s} \in \mathcal{V}, \sigma \in \mathcal{F}_\Gamma\}.$$



VAG degrees of freedom u_K, $u_\mathbf{s}$, u_σ, $u_{\mathbf{s}'}$ and VAG fluxes.



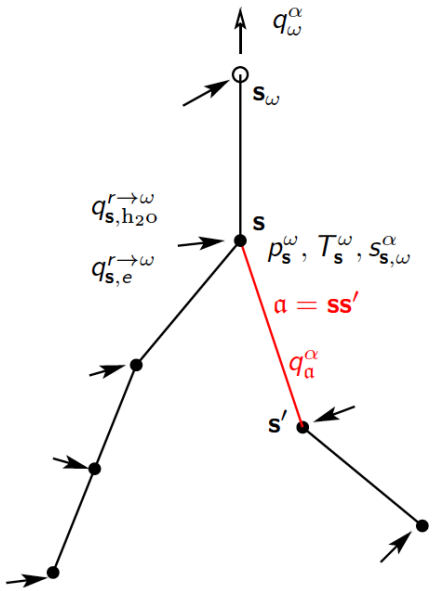
Control volumes in the case of two cells K and L.

Discretization

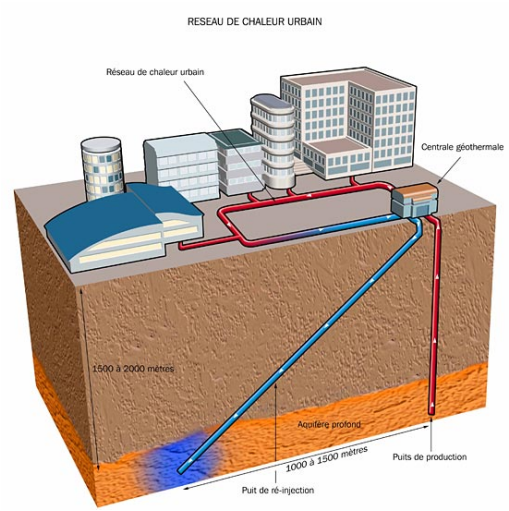
Discretization of the multi-branch wells (Notation and general setting)

Let $\mathcal{W}$ denote the set of wells.

Each multi-branch well $\omega \in \mathcal{W}$ is defined by a set of oriented edges of the mesh assumed to define a rooted tree. It is assumed that $\mathcal{V}_{\omega_1} \cap \mathcal{V}_{\omega_2} = \emptyset$ for any $\omega_1, \omega_2 \in \mathcal{W}$.



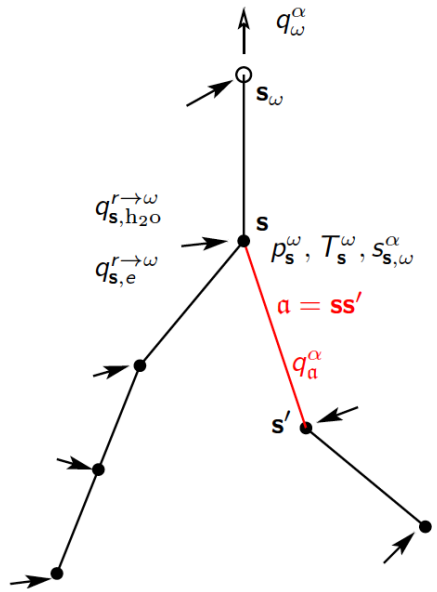
Well geometric model.



Discretization

Discretization of the multi-branch wells

- Let $\alpha \in \{\ell, \mathrm{g}\}$, **the set of well unknowns are**:
  - At each node $\mathbf{s} \in \mathcal{V}_\omega$ by the well pressure $p_{\mathbf{s}}^{\omega}$, the well temperature $T_{\mathbf{s}}^{\omega}$ and the well saturations $s_{\mathbf{s},\omega}^{\alpha}$.
  - At each edge $\mathfrak{a} \in \mathcal{E}_\omega$ by the mass flow rates $q_{\mathfrak{a}}^{\alpha}$.
  - These are complemented by the well **total** mass flow rates q_{ω}^{α} which are non negative for production wells and non positive for injection wells.



Multi-branch well ω with its root node $\mathbf{s}_\omega$, one edge $\mathfrak{a} = \mathbf{ss}'$ and the main physical quantities.

Discretization

Discretization of the multi-branch wells

- **Assumptions**:
  - The flow in the well is stationary at the reservoir time scale along with perfect mixing and thermal equilibrium.
  - The Fourier fluxes and the wall friction are neglected and the pressure distribution is assumed hydrostatic along the well.
  - Zero slip law: the velocity of the water vapor is the same as the one of water liquid inside the well.
  - There are no cross flows, i.e, the flow goes in only one direction.

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Discretization

Discretization of the multi-branch wells (Notation and general setting)

- For any $a \in \mathbb{R}$, let us define $a^+ = \max(a, 0)$ and $a^- = \min(a, 0)$. **The mass flow rates** between the reservoir and the well ω at a given **node** $\mathbf{s} \in \mathcal{V}_\omega$ are defined as follows:

$$q_{\mathbf{s},\alpha}^{r\to\omega} = \beta_\omega^{inj} \frac{\rho^\alpha(p_\mathbf{s}^\omega, T_\mathbf{s}^\omega)}{\mu^\alpha(p_\mathbf{s}^\omega, T_\mathbf{s}^\omega)} k_{r,\mathbf{s}}^\alpha(s_{\mathbf{s},\omega}^\alpha)(V_\mathbf{s}^\omega)^- + \beta_\omega^{prod} \frac{\rho^\alpha(p_\mathbf{s}, T_\mathbf{s})}{\mu^\alpha(p_\mathbf{s}, T_\mathbf{s})} k_{r,\mathbf{s}}^\alpha(s_\mathbf{s}^\alpha)(V_\mathbf{s}^\omega)^+,$$

$$q_{\mathbf{s},\mathrm{h_2o}}^{r\to\omega} = \sum_{\alpha\in\{\ell,\mathrm{g}\}} q_{\mathbf{s},\alpha}^{r\to\omega},$$

where $\beta_\omega^{prod}, \beta_\omega^{inj} \in \{0, 1\}$, $V_\mathbf{s}^\omega$ is the Darcy flux between the reservoir and the well at a given well node s obtained using the Two Point Flux Approximation

$$V_\mathbf{s}^\omega = WI_\mathbf{s}(p_\mathbf{s} - p_\mathbf{s}^\omega),$$

where $p_\mathbf{s}$ is the reservoir pressure, and $p_\mathbf{s}^\omega$ is the well pressure at node $\mathbf{s}$. $WI_\mathbf{s}$: **Peaceman well index**.

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Discretization

Discretization of the multi-branch wells (Notation and general setting)

- The **energy flow rate** is defined similarly by

$$q_{\mathbf{s},e}^{r\to\omega} = \sum_{\alpha\in\{\ell,\mathrm{g}\}} h^{\alpha}(p_{\mathbf{s}}^{\omega}, T_{\mathbf{s}}^{\omega})(q_{\mathbf{s},\alpha}^{r\to\omega})^{-} + h^{\alpha}(p_{\mathbf{s}}, T_{\mathbf{s}})(q_{\mathbf{s},\alpha}^{r\to\omega})^{+},$$

  where h^{α} is the specific enthalpy. We have assumed the heat exchanges between the well and the reservoir are **dominated by thermal convection**.

- The well coefficients β_{ω}^{inj} and β_{ω}^{prod} are used to impose specific well behavior:
  - For an **injection well**, we set $\beta_{\omega}^{inj} = 1$ and $\beta_{\omega}^{prod} = 0$. And the mass flow rates $q_{\mathbf{s},\mathrm{h_2o}}^{r\to\omega}$ are **non positive**.
  - For a **production injection well**, we set $\beta_{\omega}^{inj} = 0$ and $\beta_{\omega}^{prod} = 1$. And the mass flow rates $q_{\mathbf{s},\mathrm{h_2o}}^{r\to\omega}$ are **non negative**.

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Discretization

Recalling the equations in the reservoir for the model-H_2O-Liquid (l) and vapor (g).

- **Unknowns**: Pressure: p_m. Temperature: T_m. Saturations: s_m^l, s_m^g.
- The set of equations couples the **mass**, **energy**, **volume** balance equations and the **thermodynamic equilibrium** in the matrix (subscript $_m$):

$$
\begin{cases}
\phi_m \, \partial_t \Big(\displaystyle\sum_{\alpha \in \{\ell, \mathrm{g}\}} \rho^\alpha(p_m, T_m) s_m^\alpha \Big) + \mathrm{div}(\mathbf{q}_m^{\mathrm{h_2o}}) = 0, \\
\phi_m \, \partial_t \Big(\displaystyle\sum_{\alpha \in \{\ell, \mathrm{g}\}} \rho^\alpha(p_m, T_m) e^\alpha(p_m, T_m) s_m^\alpha \Big) + (1 - \phi_m) \partial_t E_r(p_m, T_m) + \mathrm{div}(\mathbf{q}_m^e) = 0, \\
\displaystyle\sum_{\alpha \in \{\ell, \mathrm{g}\}} s_m^\alpha = 1, \\
p_m = p_{\mathrm{sat}}(T_m) \text{ if } s_m^l > 0 \text{ and } s_m^g > 0, \\
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s_m^l = 0 \text{ if } p_m < p_{\mathrm{sat}}(T_m),
\end{cases}
$$

where ϕ_m: porosity, ρ_m: mass density, $\mathbf{q}_m^{\mathrm{h_2o}}$: mass flux, $\mathbf{q}_m^e$: energy flux, e: specific internal energy, E_r: rock energy density, and p_{sat}: the saturated pressure.

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Discretization

Two-phase flow continuous model with one component H_2O-Liquid (l) and vapor (g). No slip, no friction, no Fourier fluxes. (**Continuous Model without reservoir**)

$$
\begin{cases}
\displaystyle\sum_{\alpha\in\{l,g\}} \mathrm{div}_\tau \mathbf{q}^\alpha = \sum_{s\in\mathcal{V}_\omega} q^{r\to\omega}_{\mathbf{s},h_2o}\,\delta_{\mathbf{s}}, \\
\displaystyle\sum_{\alpha\in\{l,g\}} \mathrm{div}_\tau (h^\alpha(p^\omega,T^\omega)\mathbf{q}^\alpha) = \sum_{s\in\mathcal{V}_\omega} q^{r\to\omega}_{\mathbf{s},e}\,\delta_{\mathbf{s}}, \\
s^l_\omega + s^g_\omega = 1, \\
p^\omega = p_{\mathrm{sat}}(T^\omega) \text{ if } s^l_\omega > 0 \text{ and } s^g_\omega > 0, \\
s^g_\omega = 0 \text{ if } p^\omega > p_{\mathrm{sat}}(T^\omega), \\
s^l_\omega = 0 \text{ if } p^\omega < p_{\mathrm{sat}}(T^\omega),
\end{cases}
$$

$$
\begin{cases}
\mathbf{q}^\alpha = S_\omega \rho^\alpha(p^\omega,T^\omega) s^\alpha_\omega \mathbf{u}^\alpha, \\
\nabla_\tau p^\omega = \rho\, \mathbf{g}_\tau, \\
\rho = \rho^l s^l_\omega + \rho^g s^g_\omega, \\
\mathbf{u}^g = \mathbf{u}^l,
\end{cases}
$$

where:

τ is the spacial coordinate along the well, $\mathbf{u}^\alpha$ continuous phase velocity along the well.

$\delta_{\mathbf{s}}$ stands for the delta function at the well node s.
The fluxes with the reservoir are assumed to be concentrated at the well nodes.

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Discretization

Discretization of the multi-branch wells (General and discrete setting)

- Defining the following each edge $\mathfrak{a} = \mathbf{ss}' \in \mathcal{E}_\omega$, and each phase α:
  - The upwind approximations of the specific **enthalpy, mass density, and saturation**:

$$h_{\mathfrak{a}}^{\alpha} = \begin{cases} h^{\alpha}(p_{\mathbf{s}'}^{\omega}, T_{\mathbf{s}'}^{\omega}) \text{ if } q_{\mathfrak{a}}^{\alpha} \geq 0, \\ h^{\alpha}(p_{\mathbf{s}}^{\omega}, T_{\mathbf{s}}^{\omega}) \text{ if } q_{\mathfrak{a}}^{\alpha} < 0. \end{cases} \quad \rho_{\mathfrak{a}}^{\alpha} = \begin{cases} \rho^{\alpha}(p_{\mathbf{s}'}^{\omega}, T_{\mathbf{s}'}^{\omega}) \text{ if } q_{\mathfrak{a}}^{\alpha} \geq 0, \\ \rho^{\alpha}(p_{\mathbf{s}}^{\omega}, T_{\mathbf{s}}^{\omega}) \text{ if } q_{\mathfrak{a}}^{\alpha} < 0. \end{cases}$$

$$s_{\mathfrak{a}}^{\alpha} = \begin{cases} s_{\mathbf{s}',\omega}^{\alpha} \text{ if } q_{\mathfrak{a}}^{\alpha} \geq 0, \\ s_{\mathbf{s},\omega}^{\alpha} \text{ if } q_{\mathfrak{a}}^{\alpha} < 0. \end{cases}$$

  - Let $\kappa_{\mathfrak{a},\mathbf{s}'} = -1$ and $\kappa_{\mathfrak{a},\mathbf{s}} = 1$.
- Let $\mathcal{E}_{\mathbf{s}}^{\omega} \subset \mathcal{E}_\omega$ denote the set of well edges sharing the node $\mathbf{s} \in \mathcal{V}_\omega$.

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Discretization

Discretization of the multi-branch wells (General setting and discrete setting)

- The well equations account for the **mass and energy conservations** at each node of the well combined with **the volume balance and the thermodynamical equilibrium**. We have then the following for **each well node** $s_\omega \in \mathcal{V}_\omega$:

$$
\begin{cases}
q^{r\to\omega}_{\mathfrak{s},\mathrm{h_2o}} + \displaystyle\sum_{\mathfrak{a}\in\mathcal{E}^\omega_{\mathfrak{s}}} \sum_{\alpha\in\{\ell,\mathrm{g}\}} \kappa_{\mathfrak{a},\mathfrak{s}} q^\alpha_{\mathfrak{a}} = \delta^{\mathfrak{s}\omega}_{\mathfrak{s}} \sum_{\alpha\in\{\ell,\mathrm{g}\}} q^\alpha_\omega, \\
q^{r\to\omega}_{\mathfrak{s},e} + \displaystyle\sum_{\mathfrak{a}\in\mathcal{E}^\omega_{\mathfrak{s}}} \sum_{\alpha\in\{\ell,\mathrm{g}\}} \kappa_{\mathfrak{a},\mathfrak{s}} h^\alpha_{\mathfrak{a}} q^\alpha_{\mathfrak{a}} = \delta^{\mathfrak{s}\omega}_{\mathfrak{s}} \sum_{\alpha\in\{\ell,\mathrm{g}\}} \left(\bar{h}^\alpha_\omega (q^\alpha_\omega)^- + h^\alpha(p^\omega_{\mathfrak{s}}, T^\omega_{\mathfrak{s}})(q^\alpha_\omega)^+\right), \\
s^\ell_{\mathfrak{s},\omega} + s^{\mathrm{g}}_{\mathfrak{s},\omega} = 1, \\
p^\omega_{\mathfrak{s}} = p_{\mathrm{sat}}(T^\omega_{\mathfrak{s}}) \text{ if } s^{\mathrm{g}}_{\mathfrak{s},\omega} > 0 \text{ and } s^\ell_{\mathfrak{s},\omega} > 0, \\
p^\omega_{\mathfrak{s}} \geq p_{\mathrm{sat}}(T^\omega_{\mathfrak{s}}) \text{ if } s^{\mathrm{g}}_{\mathfrak{s},\omega} = 0, \quad p^\omega_{\mathfrak{s}} \leq p_{\mathrm{sat}}(T^\omega_{\mathfrak{s}}) \text{ if } s^\ell_{\mathfrak{s},\omega} = 1,
\end{cases}
$$

where δ stands for the Kronecker symbol, p_{sat} is the saturated pressure, and $\bar{h}^\alpha_\omega$ for prescribed specific enthalpies in the case of injection wells.

Discretization

Discretization of the multi-branch wells (General setting and discrete setting)

- Inside the well, the hypothesis of **hydrostatic pressure distribution** implies that

$$p_{\mathfrak{s}}^{\omega} - p_{\mathfrak{s}'}^{\omega} + \rho_{\mathfrak{a}} g (z_{\mathfrak{s}} - z_{\mathfrak{s}'}) = 0,$$

  where for each edge $\mathfrak{s}\mathfrak{s}' = \mathfrak{a} \in \mathcal{E}_{\omega}$, where $\rho_{\mathfrak{a}}$ is the mass density of the liquid gas mixture.

- The computation of $\rho_{\mathfrak{a}}$ is computed **explicitly**:
  - It is the variable liquid density for injectors: $\rho_{\mathfrak{a}} := \rho_{\mathfrak{a}}^{l}(p_{\mathfrak{s}}^{\omega,n-1}, T_{\mathfrak{s}}^{\omega,n-1})$.
  - It is the variable mean density for producers:

$$\rho_{\mathfrak{a}} := \sum_{\alpha \in \{\ell, \mathrm{g}\}} s_{\mathfrak{s},\omega}^{\omega,n-1} \rho_{\mathfrak{a}}^{\alpha}(p_{\mathfrak{s}}^{\omega,n-1}, T_{\mathfrak{s}}^{\omega,n-1}).$$

- The **well boundary conditions** prescribe a limit total mass flow rate $\bar{q}_{\omega}$ and a limit bottom hole pressure $\bar{p}_{\omega}$. These will be different for injector and producer wells.

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Discretization

Liquid injection wells:

- Here we have $\beta_\omega^{inj} = 1, \beta_\omega^{prod} = 0$.
- It is assumed that the injection is in liquid phase, i.e, that $s^{\ell}_{\mathbf{s},\omega} = 1, s^{\mathrm{g}}_{\mathbf{s},\omega} = 0$.
- The following are **prescribed**:

  The minimum well total mass flow rate $\bar{q}_\omega \leq 0$.
  The well maximum bottom hole pressure $\bar{p}_\omega$.
  The well specific liquid enthalpy $\bar{h}^{\ell}_\omega$.

- Given the previous assumptions, and summing all fluxes eqs. $\Longrightarrow$ **that the only implicit unknown is** $p^n_\omega = p^n_{s\omega}$ per injection well.
- The well equation at the current time step is defined by the following complementary constraints (boundary conditions at the top):

$$\begin{cases} \Big(\displaystyle\sum_{\mathbf{s}\in\mathcal{V}_\omega} q^{r\to\omega}_{\mathbf{s},\mathrm{h_2o}}(p^n_{\mathbf{s}}, p^n_\omega) - \bar{q}_\omega\Big)\Big(\bar{p}_\omega - p^n_\omega\Big) = 0, \\ \displaystyle\sum_{\mathbf{s}\in\mathcal{V}_\omega} q^{r\to\omega}_{\mathbf{s},\mathrm{h_2o}}(p^n_{\mathbf{s}}, p^n_\omega) - \bar{q}_\omega \geq 0, \\ \bar{p}_\omega - p^n_\omega \geq 0, \end{cases}$$

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Discretization

Production wells:

- Here we have $\beta_\omega^{inj} = 0$, $\beta_\omega^{prod} = 1$.
- Completely two-phase production.
- The following are **prescribed**:

  The maximum well total mass flow rate $\bar{q}_\omega \geq 0$.
  The well minimum bottom hole pressure $\bar{p}_\omega$.
- Given the previous assumptions, and summing all fluxes eqs. $\Longrightarrow$ **that the only implicit unknown is** $p_\omega^n = p_{s_\omega}^n$ per production well.
- The well equation at the current time step is defined by the following complementary constraints (boundary conditions at the top):

$$\begin{cases} \left(\bar{q}_\omega - \displaystyle\sum_{\mathbf{s}\in\mathcal{V}_\omega} q_{\mathbf{s},\mathrm{h_2o}}^{r\to\omega}(p_\mathbf{s}^n, T_\mathbf{s}^n, s_\mathbf{s}^{\ell,n}, s_\mathbf{s}^{\mathrm{g},n}, p_\omega^n)\right)\left(p_\omega^n - \bar{p}_\omega\right) = 0, \\ \bar{q}_\omega - \displaystyle\sum_{\mathbf{s}\in\mathcal{V}_\omega} q_{\mathbf{s},\mathrm{h_2o}}^{r\to\omega}(p_\mathbf{s}^n, T_\mathbf{s}^n, s_\mathbf{s}^{\ell,n}, s_\mathbf{s}^{\mathrm{g},n}, p_\omega^n) \geq 0, \\ p_\omega^n - \bar{p}_\omega \geq 0. \end{cases}$$

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Discretization

Non-Linear System I

- For each $\nu \in \mathcal{M} \cup \mathcal{F}_\Gamma \cup \mathcal{V}$ the set of reservoir pressure, temperature, saturations and mass fractions unknowns is denoted by $X_\nu = \left(P_\nu, T_\nu, s_\nu^\ell, s_\nu^{\mathrm{g}}\right)$
- We denote by $X_\mathcal{D}$, the set of reservoir unknowns

$$X_\mathcal{D} = \{X_\nu,\ \nu \in \mathcal{M} \cup \mathcal{F}_\Gamma \cup \mathcal{V}\},$$

- The set of well bottom hole pressures is denoted by $P_\mathcal{W} = \{p_\omega,\ \omega \in \mathcal{W}\}$.
- The complete non-linear system is the following:

$$\mathbf{0} = \mathcal{R}(X_\mathcal{D}, P_\mathcal{W}) := \begin{cases} R_\mathbf{s}(X_\mathcal{D}, P_\mathcal{W}),\ \mathbf{s} \in \mathcal{V}, \\ R_\sigma(X_\mathcal{D}),\ \sigma \in \mathcal{F}_\Gamma, \\ R_K(X_\mathcal{D}),\ K \in \mathcal{M}, \\ R_\omega(X_\mathcal{D}, P_\mathcal{W}),\ \omega \in \mathcal{W}. \end{cases}$$

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Discretization

Non-Linear System II

- **The reservoir equations** at each non-Dirichlet node $\mathbf{s} \in \mathcal{V}$

$$R_{\mathbf{s},i}(X^n_{\mathcal{D}}, P^n_{\mathcal{W}}) := \frac{\mathcal{A}_{i,\mathbf{s}}(X^n_{\mathbf{s}}) - \mathcal{A}_{i,\mathbf{s}}(X^{n-1}_{\mathbf{s}})}{\Delta t^n} + \sum_{\sigma \in \mathcal{F}_{\Gamma,\mathbf{s}}} -q^i_{\sigma,\mathbf{s}}(X^n_{\mathcal{D}}) + \sum_{K \in \mathcal{M}_{\mathbf{s}}} -q^i_{K,\mathbf{s}}(X^n_{\mathcal{D}}) + \sum_{\omega \in \mathcal{W} | \mathbf{s} \in \mathcal{V}_\omega} q^{r \to \omega}_{\mathbf{s},i}(X^n_{\mathbf{s}}, p^{\omega,n}_{\mathbf{s}}) = 0,$$

  where $R_{s,i},\ i \in \{\mathrm{h_2o}, e\}$.

- **The well equations**:
  - For the injection wells $\omega \in \mathcal{W}_{inj}$:

$$R_\omega(X^n_{\mathcal{D}}, P^n_{\mathcal{W}}) := -\min\Big(\sum_{\mathbf{s} \in \mathcal{V}_\omega} q^{r \to \omega}_{\mathbf{s},\mathrm{h_2o}}(X^n_{\mathbf{s}}, p^n_\omega) - \bar{q}_\omega, \bar{p}_\omega - p^n_\omega\Big) = 0.$$

  - For the production wells $\omega \in \mathcal{W}_{prod}$:

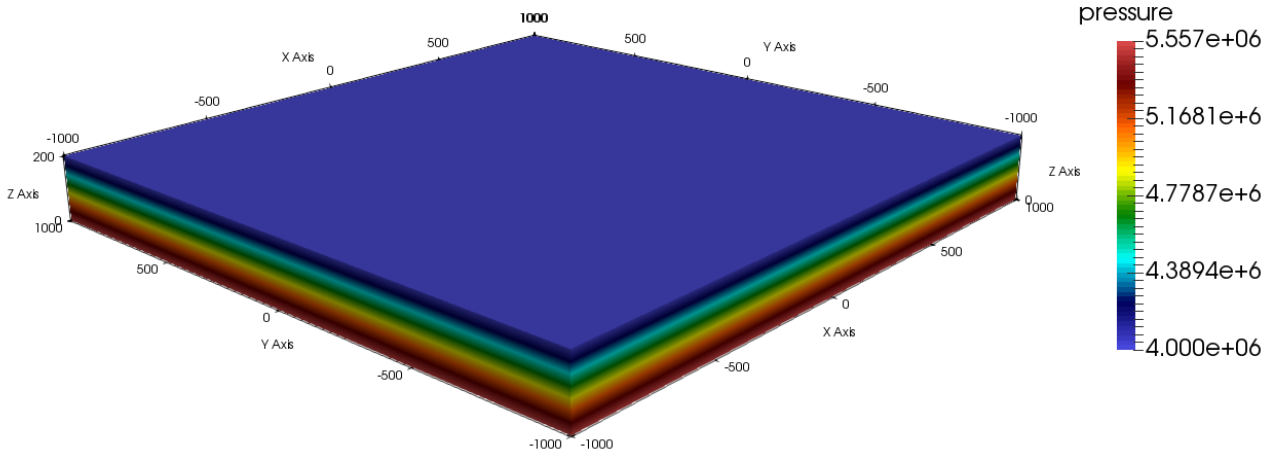
$$R_\omega(X^n_{\mathcal{D}}, P^n_{\mathcal{W}}) := \min\Big(\bar{q}_\omega - \sum_{\mathbf{s} \in \mathcal{V}_\omega} q^{r \to \omega}_{\mathbf{s},\mathrm{h_2o}}(X^n_{\mathbf{s}}, p^n_\omega), p^n_\omega - \bar{p}_\omega\Big) = 0.$$

Numerical Test

Numerical test: Diphasic vertical well

- We consider the domain $\Omega = (-H, H)^2 \times (0, H_z)$ where $H = 1000$ m and $H_z = 200$ m.
- We set one vertical producer well of radius $r_\omega = 0.1$ m, defined by the line $\{(x, y, z) \in \Omega \,|\, x = y = 0\}$.
- **The simulation consists in two stages. At the first one we have**:

  Dirichlet-BDC at the top, prescribing the pressure and the saturated temperature as a function of this pressure.
  Liquid phase in the whole domain.
  Neumann-BDC at the bottom and at the sides of the domain.
  At this stage the **well is in closed state**, i.e., nothing is produced.
  Simulation runs for 10 years.



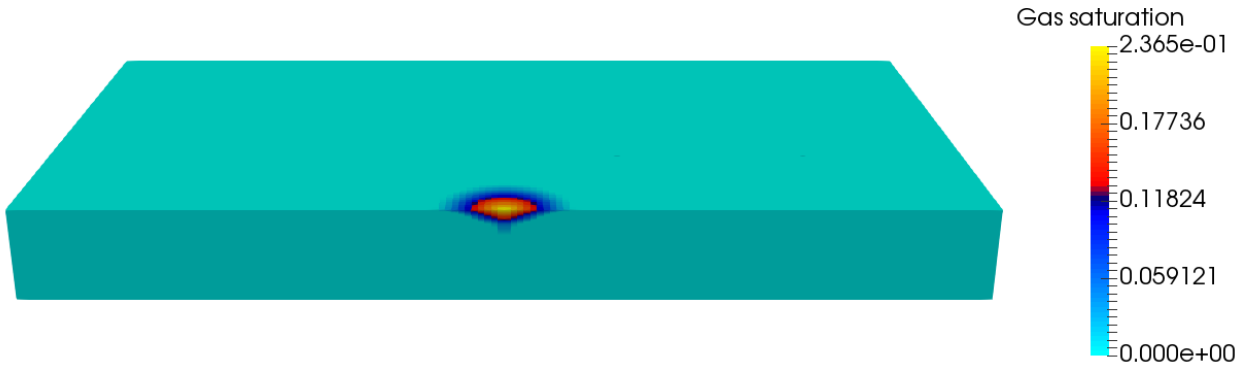
Pressure at the end of the first stage of simulation (reservoir).

Numerical Test

Numerical test: Diphasic vertical well

- **At the second stage we have**:

  Dirichlet-BDC prescribing the pressure and temperature at the sides equal to the ones obtained previously.
  Neumann-BDC at the top and at the bottom of the domain.
  At this stage the **well is in open state**, i.e., it can produce.
  Simulation runs for 30 days.

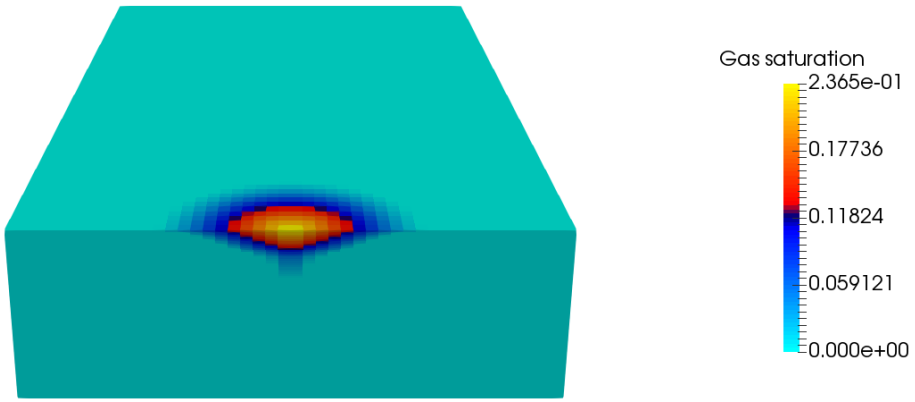


Gas Saturation cell values at the end of the simulation (domain cut at the origin).

Numerical Test

Numerical test: Diphasic vertical well

- **At the second stage:**



Gas Saturation at the end of the simulation (closer look).

- Total CPU-runtime: 6 hrs with 32 procs.
- Number of cells: 1,848,320. Number of nodes: 1,896,129.

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Numerical Test

Numerical test: Diphasic vertical well

- Computation over the sequences of meshes $\{h_1, h_2, h_3, h_4\}$.
  Convergence Test:

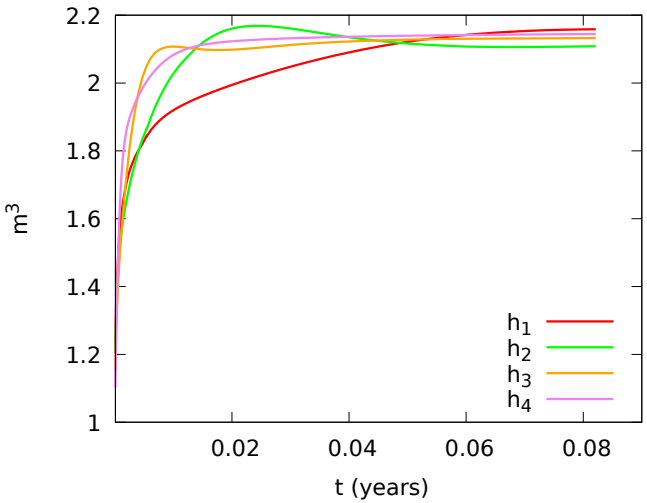

Figure: Total gas volume inside the well.

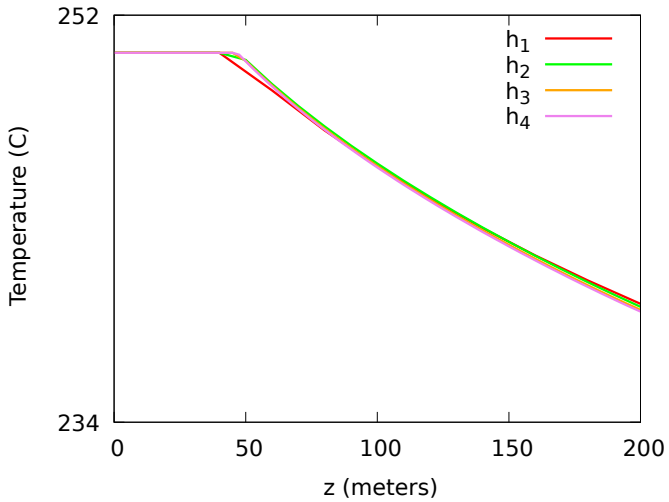

Figure: Temperature along the well.

- **Ref:** **[Armandine Les Landes, CQ, Jeannin, Lopez, Masson 2021]**

Numerical Test

Numerical test case: Doublet.

- **Application test case**: Inspired from real geological data of the region of Bouillante, Guadeloupe.
- A doublet: One injector - One producer.
- One fault.

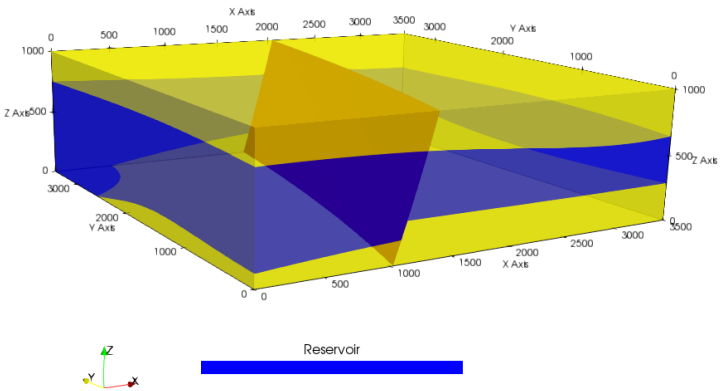

Figure: Geometry: The reservoir and the fracture.

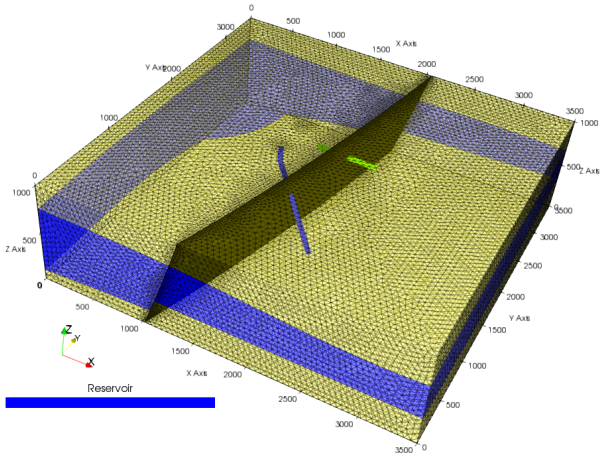

Figure: Mesh. One producer (green)/ one injector (blue).

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Numerical Test

Numerical test case: Doublet.

- **The simulation consists in two stages. At the first one we have**:
  - Dirichlet-BDC at the top, prescribing the pressure and the saturated temperature as a function of this pressure.
  - Neumann-BDC at the bottom and at the sides of the domain.
  - At this stage the wells are in closed state.
  - Simulation runs for 10^5 years.

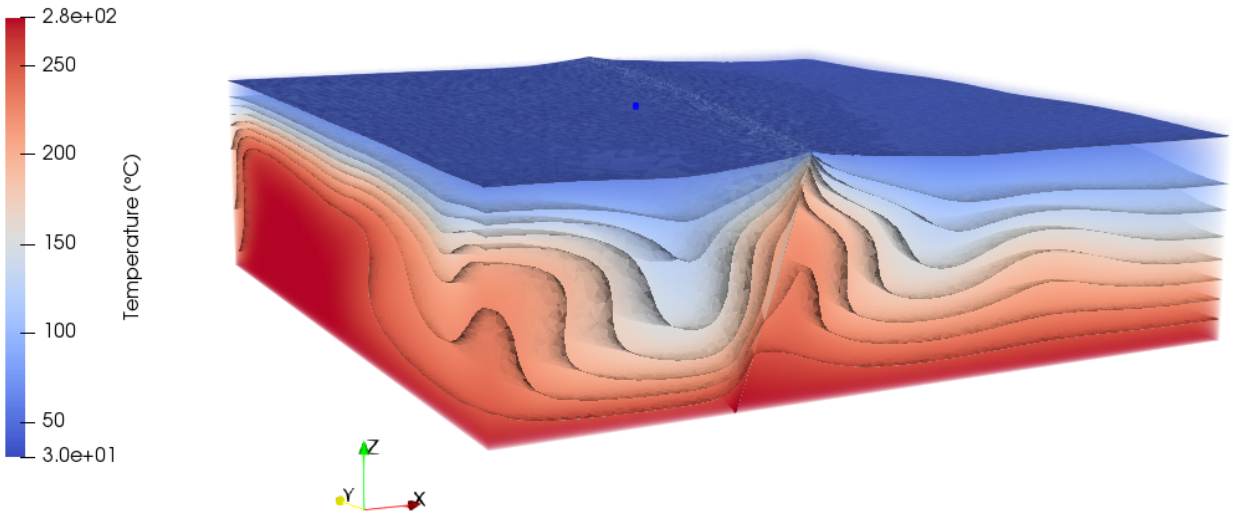

Figure: Temperature at the end of the first stage of simulation (reservoir and fault).

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Numerical Test

Numerical test case: Doublet.

- **At the second stage we have**:
  - Dirichlet-BDC prescribing the pressure and temperature at the sides equal to the ones obtained previously.
  - Neumann-BDC at the top and at the bottom of the domain.
  - At this stage only the **producer well is in open state**, i.e., it can produce.
  - After 5 years the **injector well is open**, i.e., it injects water.

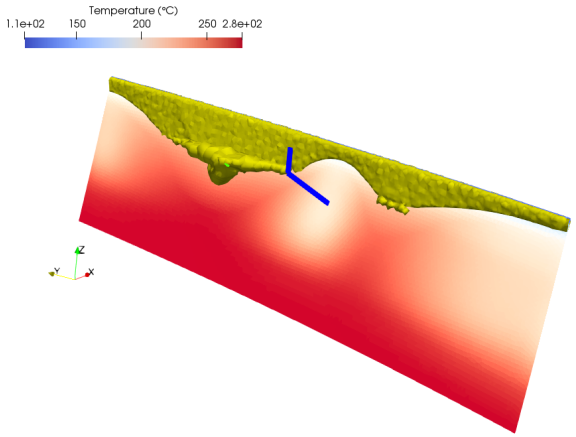

Figure: Temperature and gas saturation (in yellow) on the fault after 5 years.

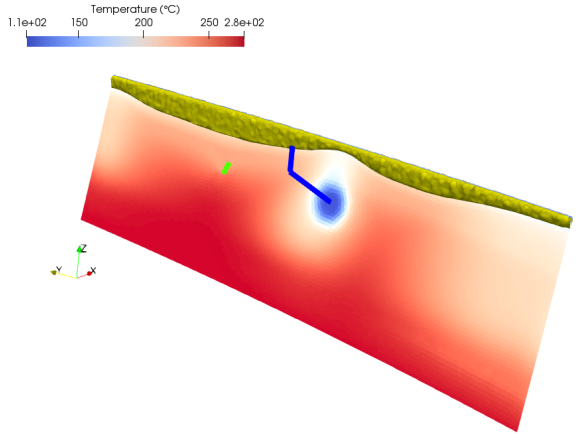

Figure: Temperature and gas saturation (in yellow) on the fault after 10 years.

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Ongoing and future work

Ongoing work

Implementing and testing a more complete model:

- Cross flows, Fourier fluxes and a non-trivial slip-law:

$$
\begin{cases}
\displaystyle\sum_{\alpha\in\{l,g\}} \mathrm{div}_\tau \mathbf{q}^\alpha = \sum_{s\in\mathcal{V}_\omega} q^{r\to\omega}_{\mathbf{s},h_2o}\,\delta_{\mathbf{s}},\ i\in C,\\
\displaystyle\sum_{\alpha\in\{l,g\}} \mathrm{div}_\tau\left(h^\alpha(p^\omega,T^\omega)\mathbf{q}^\alpha\right) + \mathrm{div}_\tau\left(-\lambda S_\omega \nabla_\tau T^\omega\right) = \sum_{s\in\mathcal{V}_\omega} q^{r\to\omega}_{\mathbf{s},e}\,\delta_{\mathbf{s}},\\
s^l_\omega + s^g_\omega = 1,\\
p^\omega = p_{\mathrm{sat}}(T^\omega) \text{ if } s^l_\omega > 0 \text{ and } s^g_\omega > 0,\\
s^g_\omega = 0 \text{ if } p^\omega > p_{\mathrm{sat}}(T^\omega),\\
s^l_\omega = 0 \text{ if } p^\omega < p_{\mathrm{sat}}(T^\omega),
\end{cases}
$$

$$
\begin{cases}
\mathbf{q}^\alpha = S_\omega \rho^\alpha(p^\omega,T^\omega) s^\alpha_\omega \mathbf{u}^\alpha,\\
\nabla_\tau p^\omega = \mathbf{T}^f + \rho\,\mathbf{g}_\tau,\\
\rho = \rho^l s^l_\omega + \rho^g s^g_\omega,\\
\mathbf{u}^g - \mathbf{u}^l = \Phi(\mathbf{u}^m,\rho^g,\rho^l,r_\omega,\theta_\omega,\ldots).
\end{cases}
$$

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References I

References


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